

## Reaction times as a measure of uncertainty

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In this article, a method is presented to analyse relationships between detection or discrimination frequencies and reaction times in psychophysical tasks. It is shown in three empirical data sets that reaction time decreases as a linear function of the absolute value of the logit transforms of the response probabilities. Such a function stresses the characteristic uncertainty associated with subjects' responses and the relation between their response parameters and the response criterion used by the subjects.

*El tiempo de reacción como una medida de incertidumbre.* En este artículo se presenta un método para analizar las relaciones entre las frecuencias de detección o discriminación y el tiempo de reacción en tareas psicofísicas. A partir de tres conjuntos de datos empíricos se demuestra que el tiempo de reacción disminuye según una función lineal del valor absoluto del logit de las probabilidades de respuesta. Esta función pone de relevancia la incertidumbre característica asociada a las respuestas de los sujetos y la relación entre sus parámetros y el criterio de respuesta empleado por éstos.

Piéron's law is the most general relationship found between reaction time and intensity of a stimulus (Piéron, 1914, 1920, 1952). It reflects a response mechanism underlying most of the reaction time models. A response mechanism represents the component of the model which estimates the relative weight of the different alternatives and selects the most relevant one. In most of the models, the response mechanism is no more than an activation threshold for response production. The role of the response mechanism may be elucidated by examining the functional relation between the reaction time and the strength of evidence (Stafford & Gurney, 2005). In this sense, Piéron's law states that, at a constant intensity of the background (Hsu, 2005), reaction time decreases as a power function of the intensity of the target stimulus according to  $RT = (\beta I^{-\alpha}) + t_0$ ; where RT is the reaction time,  $\beta$  is an adjusting parameter,  $\alpha$  is the exponent of the function that represents a parameter of sensitivity characteristic of each sensorial system (Bonnet, Zamora, Buratti, & Guirao, 1999), and  $t_0$  is the asymptotic reaction time. The value of this later parameter depends on sensory and decisional factors (Bonnet & Dresch, 2001). Piéron's law applies to simple as well as choice reaction time experiments (Pins & Bonnet, 1996) for suprathreshold intensities where response probability is 1 (or close to 1). In this case, it was shown (Link and Bonnet, 1998) that individual Piéron's functions are parallel so that individual  $t_0$ , or the mean reaction times, reflects differences in decision criteria between subjects (Bonnet & Dresch, 2001).

In the threshold region, where response probability varies with stimulus intensity, reaction time decreases also as a hyperbolic

function of the intensity of the stimulus. Pins and Bonnet (2000) have shown that Piéron's law also applies in this case. However, the exponent of the function is higher when intensities are in the threshold range than when they are in the suprathreshold range. This is due to the fact that response uncertainty adds its effects to those of intensity.

We developed further Link's model (Link, 1992) for experiments in which reaction time is measured as a function of the intensity of suprathreshold stimuli (Bonnet & Link, 1998; Link & Bonnet, 1998; Bonnet & Dresch, 2001). Link's model is a random walk representation (Laming, 1968; Link, 1975; Link & Heath, 1975; Stone, 1960) which belongs to sequential sampling models which constitute dominant dynamic models of psychophysical decisions (Heathcote, 1998; Luce, 1986). These models are based on the assumption that stimulus representation in the nervous system is noisy. Hence, in order to make a decision upon the stimulus, the system should accumulate information by successive sampling until a given level of evidence or information criterion is reached (Bonnet & Link, 1998; Link & Bonnet, 1998; Maiche, Fauquet, Estaún, & Bonnet, 2004; Ratcliff & Smith, 2004). In these models, the stimulus's evidence for a response is compared by difference to the other response.

In many detection or discrimination experiments, reaction time is measured together with choice frequencies. With his Wave Theory of discrimination, Link (1992) proposed a model handling these two responses in discrimination experiments, where the word «wave» is «a vivid metaphor for the stochastic process that describes the temporal unfolding of the stimulus representation in the perceptual system» (Smith, 1994, p. 409). The model assumes that response probabilities and reaction times depend basically on the following parameters.  $\mu$  measures the rate of accumulation of information and is linearly related to the stimulus intensity differences ( $\Delta s$ ) according to  $\mu = \alpha \Delta s + \beta$ . The psychometric function is a logistic function (Link, 1978) assuming the

responsiveness ( $A$ ) constant within the experiment; in this sense,  $p = 1/(1 + e^{-\theta A})$  so that  $\text{logit}(p) = \ln[p/(1-p)] = \theta A$  which is linear.  $\theta$  is a discrimination parameter which affects response probability and relates the mean  $\mu$  and higher moments of the probability distribution; and  $A$  which represents the response threshold of the subject or responsiveness. The later is assumed to be constant for a given subject in a given experiment. In this sense, given that  $\theta$  is a function of stimulus intensities,  $A$  could be estimate by least square method from the slope of the line that relates the logits to stimulus intensities. Similarly, Palmer, Huk and Shadlen (2005) define  $p$  incorporating the stimulus strength into the exponent of the above exponential function.

The chronometric function express the reaction time as a function of the stimulus intensities or differences or, in term of the model the number of samples before responding or decisional time (DT) and the values of  $\mu$  according to  $DT = A \times (2p - 1) / \mu$ . Finally, a linear function relates the mean reaction times to a variable  $Z$  defined in terms of probability of response and which relates to discrimination and presumably to detection and combines 4 parameters. From a practical point of view, while the logistic psychometric function is easily fitted to experimental results, the chronometric function is more complex to use, as well as the reaction time versus  $Z$  relation.

Many other solutions have been proposed to describe such a trade-off between speed and accuracy (i.e. Luce, 1986; Palmer et al., 2005), whether accuracy is estimated with  $d'$  or through a psychometric function. The aim of the present paper is to present an empirical solution to relate reaction time to stimulus probabilities (accuracy) whether we consider detection or discrimination thresholds. The psychometric function relating the probability of response in a detection or in a discrimination experiment is assumed to be a logistic function of the intensities. In a forced choice detection experiment, reaction time decreases monotonically when intensities increase following a Piéron's function; in discrimination experiment or in a yes/no detection experiment, reaction time varies curvilinearly with the stimulus differences, being larger around the intensity giving a response probability of 0.5 (point of subjective equality in discrimination, threshold in a yes/no detection).

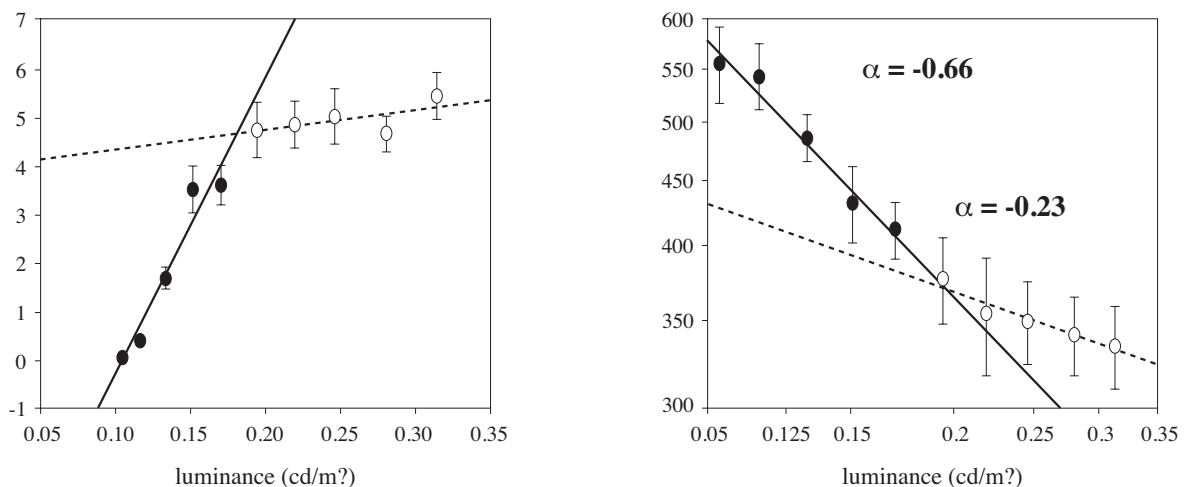
In threshold experiments, the fit of Piéron's functions which contains 3 parameters may be hazardous because of the small size of the samples. The study of the relationship between RT and response accuracy is necessary. We proposed here a simple solution to express this speed-accuracy function (SAF). First, a logit transform of the response probabilities allows linearizing the psychometric function. Second, the function relating the absolute values of the logits of the response probabilities to the corresponding reaction time is estimated. The use of the absolute values of  $\text{logit}(p)$  assumes that the psychometric function is symmetrical around the point of maximum uncertainty, that is if  $p = 0.5$  then  $\text{logit}(p) = 0$ . Such a function should be linear in the form  $RT = \beta - \alpha |\text{logit}(p)|$ . The interpretation of the two parameters ( $\beta$  and  $\alpha$ ) will be tentatively proposed.

In order to verify the empirical prediction of a such a linear function (SAF), some data are reanalyzed. Two of our own experiments were introduced in order to allow individual analysis of the results.

#### Experiment 1: detection in 2AFC paradigm

Data from Pins and Bonnet (2000) were reanalyzed in this way. In a first experiment (exp. 1a), subjects had to decide in a 2AFC paradigm whether a small luminous square appeared at the left or at the right of the fixation point. Ten luminance levels of the luminous square were randomly presented between 0.10 to 0.31  $\text{cd/m}^2$ . The lower levels were expected to be in the threshold region, while the upper ones were expected to be above it. However, the limit between these two regions was not known a priori. Four well trained subjects took part in the experiment. Instructions request the subject to respond as fast as possible avoiding errors.

In figure 1 (left), logit transform of the response probabilities is plotted as a function of luminance levels. Clearly, a single linear function does not fit these results satisfactorily. The figure shows that in order to fit the relationship between the response probabilities to the stimulus intensities two psychometric functions are needed. The two linear functions were fitted on the basis of a least square criterion to sort the results in two ranges: the



**Figure 1.** Experiment 1a: psychometric functions (left;  $\text{logit}(p)$  vs. luminance) and chronometric functions (right; in log coordinates with the exponents,  $\alpha$ , of Piéron's functions). Dark symbols for the data falling in the threshold region, open symbols for the data falling in the suprathreshold region

threshold region (lower range) and the above threshold region (upper range). Their limit was not decided a priori, but corresponds to the best fit criterion for two functions. The lower range shows a typical psychometric function. The theoretical function for the upper range should be horizontal, but likely due to time pressure, 1% to 2% errors are left (detection probabilities are greater than 0.98).

Now, since reaction time was also measured in this experiment, Figure 1 (right) plots the mean reaction times, error and correct responses included, as a function of stimulus intensities on logarithmic coordinates. Piéron's functions were fitted to the results. Again two functions with a different exponent are needed for a satisfactory fit. A least square criterion was used to distinguish two ranges of intensities. It should be mentioned that the limit of the two ranges at which the two functions intersect is very similar for  $\logit(p)$  and reaction time while the fittings were done separately on the two variables. As was found before (Bonnet & Dresch, 2001), the exponent  $\alpha$  of Piéron's function is higher in the threshold region ( $\alpha = -0.66$ ) than in the suprathreshold region ( $\alpha = -0.23$ ).

In order to gather more data in the threshold region, a second control experiment (exp. 1b) was run in the same conditions on 3 other well trained subjects in restricting the 6 luminance levels to the threshold region defined from the previous experiment, namely between 0.13 and 0.15 cd/m<sup>2</sup>. As above, instructions request the subject to respond as fast as possible avoiding errors.

The relationship between reaction time and  $\logit(p)$  in the threshold region was studied separately for the two experiments. Figure 2 shows the expected linear relationship. While the levels of luminance, the range of these levels and the subjects are different, the slopes are very similar. The goodness of the fits were quite good with  $r^2 = 0.98$  for the first experiment and  $r^2 = 0.96$  for the second one. A small difference in intercepts is observed. These data confirm that reaction time is linearly related to  $\logit(p)$  in the threshold region of intensities as predicted by the SAF.

In order to help to interpret the parameters of the SAF, correlations between individual slopes of the SAF (both experiments mixed) were correlated with other statistics of the results (threshold, slope of the psychometric function etc.). With the exception of one subject, the only significant correlation was found between slopes of the individual SAF and the mean RT of each subject ( $r = 0.94$ ,  $p < 0.01$ ).

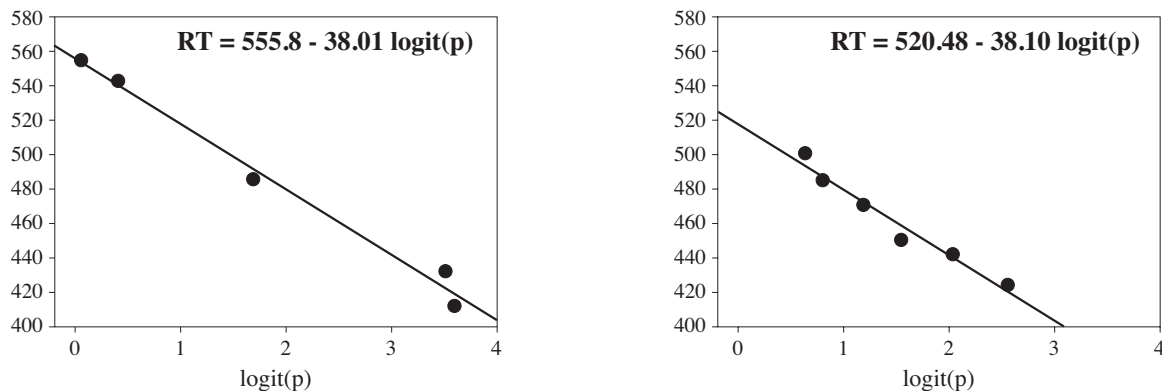
## Experiment 2: classical luminance discrimination

Since Link (1992) illustrated its linear reaction times versus Z function with Kellogg's results (1931), it is worthwhile to show that our solution applies as well to these data. In this sense, Kellogg's study stands out as exceptional because the subjects received practice extensive by making some 4000 comparisons before the experimental data were gathered, the experimenter gathered both response probabilities and reaction times and, finally, the experimental design applied allows to take on constant threshold. The fit of the linear function SAF for these discrimination data is shown in Figure 3. On the left the psychometric logistic function of the relation between response probabilities and luminance differences was fitted to the results. Figure 3 on the right shows how reaction time varies as a function of luminance differences: longest reaction time happens for the luminance difference leading to the largest uncertainty. Due to the small number of luminance differences, fits of Piéron's functions would be hazardous.

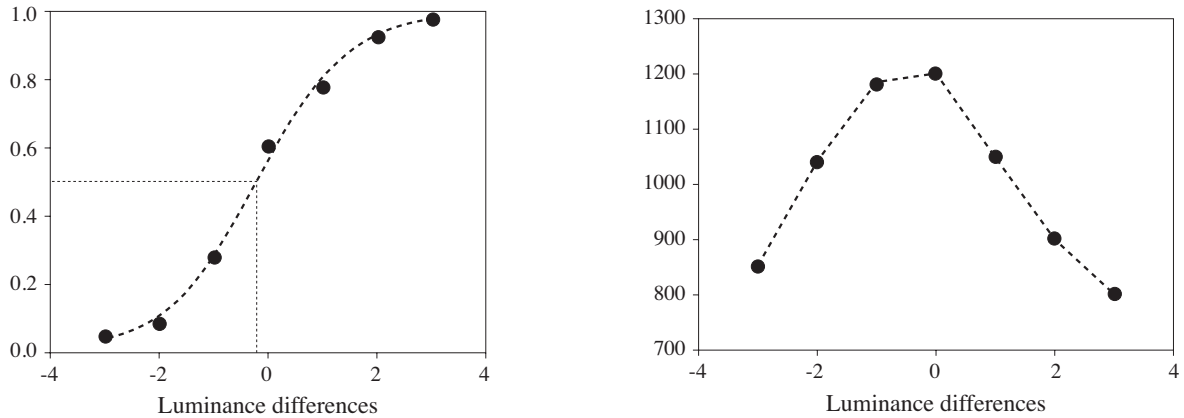
Assuming that the chronometric functions for the range of negative and positive luminance differences are symmetrical, we have fitted the relationship between RT and the absolute values of  $\logit(p)$ . In doing this, we avoid to have two functions, one for the negative logit values and one for the positive ones. Figure 4 shows that the relation between probabilities and reaction times is well fitted by a linear function  $RT = \beta - \alpha |\logit(p)|$  with  $r^2 = 0.91$ . On Kellogg's results, the fit of a SAF is as good as the reaction times versus Z Link's function.

## Experiment 3: discrimination of postures

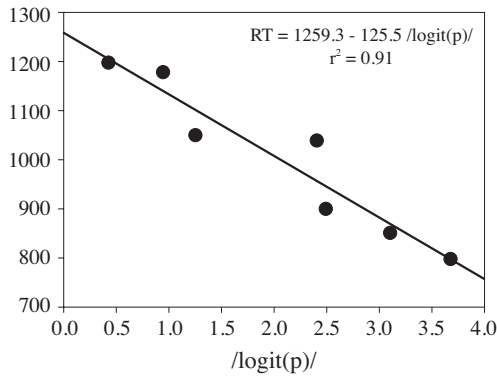
The last example of the fit of the function on discrimination results of a more cognitive task (Bonnet, Paulos, & Nithart, 2005) will allow to speculate about the meaning of the slope  $\alpha$  of the function. In this experiment, 10 naïve subjects were shown with shortly presented pictures of a human body, a mannequin or a skeleton. Subjects had to decide whether the figure was in balance or falling. The upper part of the body figure was slanted either in a forward or in a backward direction. Similar angles of body slants were used for forward slant and for backward slant (0°, 20°, 40° or 60°). Instructions request the subject to respond as fast as and as precisely as possible whether the figure was on fall or on



**Figure 2.** The linear relationship between  $\logit(p)$  and reaction time (RT) in the threshold range for the first detection experiment 1a (left) and the second experiment 1b (right) at the same scale



**Figure 3.** Logistic psychometric function of Kellogg's discrimination data (left). Reaction Times (RT) for the same luminance differences (right)



**Figure 4.** Fits of the function  $RT = \beta - \alpha / \logit(p)$  to the discrimination data Kellogg's (1931)

balance. The same type of analysis as above was applied to the results of this discrimination experiment.

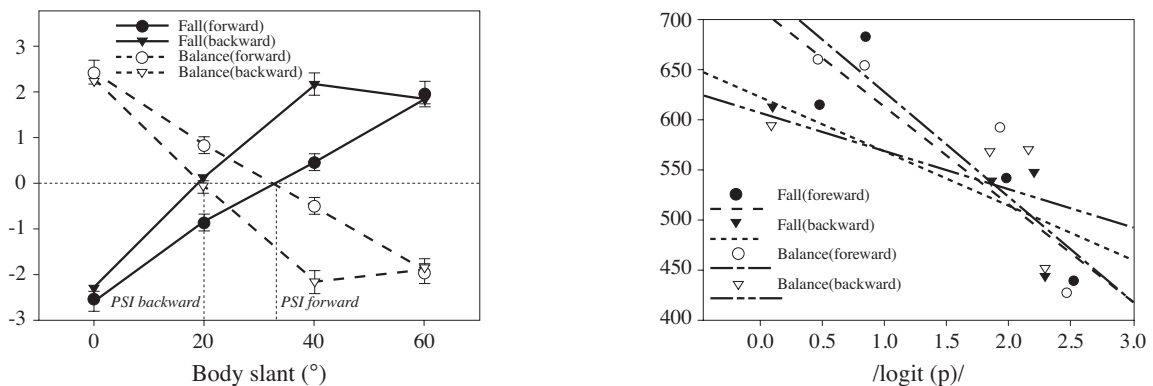
Figure 5 (left) presents the psychometric functions in logit units separately for conditions in which the figure was slanted forward or backward and for the fall and balance responses. For sake of clarity, results have been pooled for the three figures (human body,

mannequin and skeleton) which gave very similar results. For each function, we estimated a Point of Subjective Imbalance (PSI) which represents the slant angle at which the figure is equally often judged in fall or in balance. It corresponds to a smaller slant backward ( $\approx 20^\circ$ ) than forward ( $\approx 33^\circ$ ).

In the chronometric functions (not shown), reaction time is longer for the slant angle closer to the PSI, i.e. when the uncertainty is maximum. Hence the maximum appears at a lower slant for backward responses than for forward responses. Reaction times are systematically shorter for backward figures than for forward ones, except for the  $0^\circ$  slant at which all reaction times are about equal. Reaction times for balance and fall responses are very similar.

Figure 5 (right) presents the SAF relating reaction time RT and the absolute values of  $\logit(p)$ . As for experiment 3, it is assumed then that the RT of the fall and balance responses are symmetrical around the PSI. Consequently, the linear function fitted to the data is  $RT = \beta - \alpha |\logit(p)|$ , where  $|\logit(p)|$  stands for the absolute values of  $\logit(p)$ .

As explained in another paper (Bonnet et al., 2005), subjects judged the fall more likely to happen at a smaller slant angle for a backward slant than for a forward one. As expected, the differences in the slopes of the SAF functions are negligible



**Figure 5.** Psychometric functions for fall (dark symbols) and balance responses (open symbols). Standard errors between stimuli are shown (left). Linear fits of the relationship between  $\logit(p)$  (in absolute values) and RT (right). (Fall responses:  $\bullet$  forward slant,  $\blacktriangledown$  backward slant; Balance responses:  $\circ$  forward slant,  $\triangle$  backward slant)

regarding the type of responses (fall vs. balance), which confirms that the psychometric and chronometric functions are symmetrical around the PSI. Figure 3 (right) shows the SAF functions separately for the two slant directions. The averaged slope is larger for a forward slant ( $r^2=0.95$ ) than for a backward one ( $r^2=0.82$ ).

Further analysis were done on individual results. First of all, correlations between backward and forward data were computed. The best correlation was found for the mean RT ( $r=0.95$ ,  $p<0.01$ ). While, the Points of Subjective Imbalance differ between the two conditions, the slopes of the psychometric functions correlate ( $r=0.62$ ,  $p<0.5$ ). These two results suggest a good consistency of the individual decision criteria between the two conditions. Second, correlating the different statistics within each condition confirms the result of experiment 1: the best correlation is observed between the slope of the SAF function and the mean RT with  $r=0.81$  for backward condition and  $r=0.73$  for forward condition ( $p\leq 0.01$ ).

### Discussion

A simple linear relationship (SAF) between reaction times and response frequencies (their logit transform) is shown here on three examples. That relation applies both to detection threshold experiments and to discrimination experiments. In both cases, response frequencies vary as a logistic function of the intensity or difference of the stimulus (Link, 1978). Generally, reaction time varies as a Piéron's function with the intensity of the stimulus or more precisely with the uncertainty of the responses: it is maximum when the uncertainty is maximum, i.e. at the threshold value in detection experiment or at the Point of Subjective Equality in discrimination experiment (or PSI in the present example). An empirical way to rely simply RT to the certainty of the response is to take the absolute value of the logit transform of the responses probabilities  $RT = \beta - \alpha |\logit(p)|$ .

The intercept  $\beta$  of the function is the longest *reaction time* observed when the uncertainty is maximum. In the third experiment, it is observed that forward and backward slants lead to different slopes of the function: subjects are willing to respond more frequently (and faster) «fall» (vs. «imbalance») for a backward slant than for a forward one. This difference in slopes results from a combined effect of a smaller PSI and faster RTs in backward fall conditions. Tentatively, it may be suggest that, in their response decision, subjects take correctly into account the fact that the likelihood of a fall is greater in the backward direction than in the forward one since in the later case a forward motion of one leg can more easily prevent the fall. As we expected in that experiment, subjects judged the likelihood of a fall in the cognitive context of a human body representation. In a control experiment (Bonnet et al., 2005) using an unanimated object as figure, no imbalance between forward and backward falls was observed.

The fit of the SAF is satisfactorily on the group mean data. It also holds on individual results but exhibits a large interindividual variability in the estimates of its parameters. It can be evaluated in comparing the interindividual variability of the threshold or of the PSI and of the mean reaction time. In this sense, and from descriptive approach, coefficient of variation expressed in percents was calculated (see table 1 in appendix) as the ratio of the standard error of the mean to the mean expressed in percents. Interindividual performances are clearly less variable between subjects of experiment 1 than between subjects of experiment 3 in spite of the fact that the subjects of the two experiments 1a and 1b were pooled.

In order to understand better which relation the parameters of the SAF can reveal between sensory accuracy and decision criteria we have studied the correlations between intercept and slope of the SAF and of the logistic functions, threshold or PSI, and mean RT in experiment 1 and 3. For experiment 3, data of the forward and of the backward conditions have been analyzed separately. In the three occasions, as usual, the slope and the intercept for the SAF and for the logistic function present, descriptively, a high correlation. None of the correlation between the slope and the intercept of the SAF and any of the parameter of the logistic functions approached significance. Finally, in the three set of data, the slope and the intercept of the SAF correlate with the mean reaction time. If, with caution, the slope of the logistic function is considered as a valid accuracy index, these analyses would suggest that the individual parameters of the SAF reflect individual differences in decision criteria.

A further argument for the later interpretation is found in the comparison of variability of the slopes of the SAF and of the logit functions. The later should be smaller than the former since all of our subjects have a normal or corrected to normal vision. Moreover, the interindividual variability in experiment 1 should be smaller than in experiment 3, since the former used well trained subjects (see table 1 in appendix).

Many speed-accuracy trade-off functions have been proposed (Luce, 1986; Usher, Olami, & McClelland, 2002; Marcos & Redondo, 2005; Palmer et al., 2005; McMillen & Holmes, 2006; Rinberg, Koulakov, & Gelperin, 2006) mostly for experiments in which time pressure is varied in different ways. Notwithstanding the way subjects are trading accuracy for speed, any interpretation of reaction times became difficult. More, the complexity of the trading relation may oblige to correct reaction times for different conditions in taking into account the related percentage of errors. Then, as reaction time can be determined, at least hypothetically, by more than one process, from a biological perspective, more than one tradoff function is likely to exist (Luce, 1986; Osman, Lou, Muller-Gethman, Rinkenauer, Mattes, & Ulrico, 2000; Smith & Ratcliff, 2004). The main reaction time models proposed in the literature attempted to explain both errors and reaction times within a single mechanism. Consequently, such models are controversial, complex and likely valid only in limited conditions (Ratcliff & Smith, 2004). The most sophisticated reaction time models try to coordinate three analysis levels: the rate of individual neuron firing, the statistical properties of a neuron population and the behavioral data. The present work concerns the analysis of the later aspect. For this, the experiments reported here used traditional instructions suggesting an equal weight to speed and accuracy. Taken together, the present results provide a good evidence for the generality of the proposed relationship between reaction times and response probabilities; and at least in this case, the SAF is an empirical solution which fits nicely the results.

Appendix  
Table 1  
Coefficients of variation (in percent)

	Exp. 1	Exp. 3a	Exp. 3b
Th (or PSI)	5.0	16.2	12.5
Mean RT	3.3	11.6	11.3
SAF	13.8	25.4	32.3
Logit	10.5	9.8	27.3



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